A Three Part Constitutive Model for Passive Anisotropic Tissue

Catherine A. O’Connor1, David R. Nolan2, Eóin McEvoy1, Patrick McGarry1
1Department of Biomedical Engineering, National University of Ireland, Galway
2Trinity Centre for Bioengineering, Trinity Biomedical Sciences Institute, Trinity College, Dublin
c.oconnor2@nuigalway.ie

Abstract

In this paper mechanical testing of ovine aorta is performed to characterize the anisotropic behavior of aortic tissue. Results suggest that exponential strain hardening formulations previously developed for “muscular” arteries are not suitable for “elastic” arteries. A quasi-bilinear anisotropic hyperelastic formulation is developed and demonstrates that this new material model is capable of accurately simulating our experimental test data.

1. Introduction

Several hyperelastic formulations for soft tissue use an exponential form of the strain energy density to simulate strain stiffening due to extension of embedded collagen fibers [1, 2, 3]. Such fiber based formulations also incorporate in-plane fiber dispersion [4], in addition to out-of-plane fiber dispersion [5]. Such anisotropic exponential strain stiffening hyperelastic laws with two fiber families have been shown to accurately simulate muscular arteries, e.g. iliac [4, 6] and coronary [7] arteries.

2. Model Development

Building upon previous constitutive laws for soft tissue we use an additive decomposition of the strain energy density function into an isotropic component, representing two families of reinforcing fibers

\[ \psi = \psi_{iso} + \sum_{i=4,6} \psi_i \]

For simplicity we assume that the ground matrix matrix behaves as a slightly compressible neo-Hookean material [8, 9]. We propose an anisotropic component that consists of three distinct regimes (Figure 1A): (i) an initial compliant quasi-linear region to model the stretching of coiled collagen fibers; (ii) a non-linear transition region to model the uncoiling of fibers; (iii) a stiff quasi-linear region to model fully uncoiled fibers.

The anisotropic strain-energy density function is

\[ \psi_a = \begin{cases} \sum_{i=4,6} K_i I_i + 2K_i I_i^{-2} + \psi_0 & (\varepsilon_i) \leq D_1 \\ \sum_{i=4,6} T_i I_i + T_i I_i^{-2} - T_i I_i^{-2} + \psi_0 & D_1 \leq (\varepsilon_i) \leq D_2 \\ \sum_{i=4,6} K_i I_i - 2I_i^{-2} + \psi_0 & (\varepsilon_i) \geq D_2 \end{cases} \]

where \( \varepsilon_i \) is the axial strain in a fiber and parameters \( D_1 \) and \( D_2 \) define the boundaries of the three fiber deformation regimes, as shown in Figure 1A. \( K_i \) and \( K_2 \) govern the effective stiffness of the two quasi-linear regions. Two fiber families are considered; \( i=4 \) and \( i=6 \). The anisotropic invariant \( I_i \) is the square of the stretch of fibre \( i \), given as

\[ I_i = C : M_i \otimes M_i \]

where \( C \) is the right Cauchy-Green deformation tensor and \( M_i = FM_i \), where \( F \) is the deformation gradient and \( M_i \) is a vector describing the undeformed orientation of fibre \( i \). The Cauchy stress is obtained from the strain energy density function using

\[ \sigma = J^{-1} \frac{\partial \psi}{\partial F} \]

where \( J=\text{det}(F) \).

Fiber dispersion, as illustrated in Figure 1B is incorporated into the constitutive model through the generalized structure tensor \( H \) proposed by Holzapfel et al. [5], whereby

\[ H = AI_1 + BM_i \otimes M_i + C_{op} M_n \otimes M_n \]

where \( M_n \) is a vector orthogonal to the plane formed by the two fiber families. Parameters \( A \) and \( B \) are functions of the in-plane and out-of-plane fiber dispersions and \( C_{op} = (1-3A-B) \). The dispersed fibre strain for family \( i \) is given as

\[ E_i = H_i (C - I) \]

leading to

\[ E_i = AI_1 + BI_1 + C_{op} I_n \]

Fiber dispersion is then incorporated into the anisotropic strain energy density function by substitution of \( E_i \) for \( I_i \) in equation (2). The Cauchy stress, obtained from equation (4) is then given as

\[ \sigma = J^{-1} \frac{\partial \psi}{\partial F} \]

This anisotropic hyperelastic formulation is implemented in a user defined material subroutine (UMAT) in Abaqus 6.14 (Simula, Providence, RI, USA).
In order to illustrate the importance of fiber dispersion in our new formulation we simulate stretching of aortic tissue in the 1-direction, with the tissue free to expand or contract in the lateral directions ($\sigma_{22} = \sigma_{33} = 0$). In Figure 2 we show that an auxetic expansion in the out-of-plane direction, $\varepsilon_3 > 0$, occurs for high ratios of fiber stiffness to matrix stiffness ($R$) when dispersion is not included in the model. Such auxetic behavior is not observed in our experiments on aortic tissue and has not been reported in any previous mechanical tests on arterial tissue. When dispersion is included in the model the out-of-plane dispersion parameter $C_{\text{og}}$ must be positive in order to ensure that $\varepsilon_3 < 0$.

Figure 2: Out-of-plane strain $\varepsilon_3$ as a function of the applied strain $\varepsilon_1$. Note that $\varepsilon_3 > 0$ signifies auxetic behavior, which is not observed experimentally for aortic tissue. $R$ is the ratio of fiber stiffness to matrix stiffness.

3. Experimental Testing

Dog-bone specimens of descending ovine aorta in the axial ($n=8$) and circumferential ($n=9$) directions were subjected to uniaxial tensile testing to failure at a nominal strain rate of 0.0125\(^{-1}\).

Figure 3 shows the nominal stress-strain behavior (mean±SD) of the aortic tissue when stretched in the circumferential and axial directions. Clearly the tissue is highly anisotropic for the entire applied strain range, with the circumferential direction exhibiting a significantly higher stiffness. The three regions of deformation outlined in Figure 1A are clearly evident. In the case of the circumferential direction the transition region between the two linear regions is observed in the strain range from 0.65 to 0.75. A quasi-linear stress-strain relationship is observed before and after the transition region. The proposed piecewise hyperelastic model provides a good prediction of the material anisotropy, particularly in the transition and stiff quasi-linear regimes.

Figure 3: Experimental test results (mean±SD) for stretching of the aortic tissue in the circumferential and axial directions. Model predictions are also shown.

4. Discussion

Experimental testing of anisotropic aortic tissue to high strains reveals that the tissue exhibits three deformation regimes: a low stiffness linear regime; a non-linear transition regime; a high stiffness linear regime. A novel hyperelastic formulation is proposed to model this behavior. Incorporation of fiber dispersion is required to eliminate non-physical auxetic behavior. An accurate model of aortic hyperelasticity is critical for the design of implant grafts for the treatment of aortic aneurysms where a key challenge is the maintenance of physiological levels of effective vessel compliance.

5. References


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